Chapter 5 - Day 1

Derivative of a Constant Function

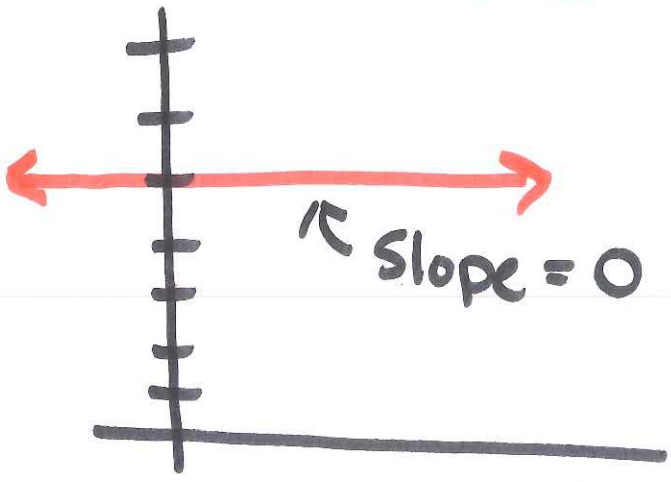
if $f(x) = c$, then $f'(x) = 0$

or $\frac{d}{dx}(c) = 0$

Ex: let $f(x) = 5$ find $f'(x)$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5 - 5}{h}$$

$$= \lim_{h \to 0} 0 = \boxed{0}$$



Power Rule

if $f(x) = x^n$ then $f'(x) = n x^{n-1}$

or $\frac{d}{dx}(x^n) = n x^{n-1}$

* best news - this works for any exponent

Ex: find $f'(x)$ for $f(x) = x^3$

$f'(x) = 3x^{3-1} = 3x^2$

Ex: find $g'(x)$ for $g(x) = \sqrt[3]{x+1}$

$g(x) = (x+1)^{1/3}$

$g'(x) = \frac{1}{3}(x+1)^{1/3-1} = \frac{1}{3}(x+1)^{-2/3}$

$= \frac{1}{3(x+1)^{2/3}}$

Ex: find $f'(t)$ for $f(t) = t^{-4}$

$$f'(t) = -4t^{-4-1} = -4t^{-5}$$

Ex: find $g'(x)$ for $g(x) = \frac{1}{\sqrt[7]{x}}$

$$g(x) = \frac{1}{x^{1/7}} = x^{-1/7}$$

so

$$g'(x) = \frac{-1}{7} x^{-1/7-1} = \frac{-1}{7} x^{-8/7}$$

The Constant Multiple Rule

$$(cf(x))' = c(f'(x))$$

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

Ex: find $f'(x)$ for $f(x) = 2x^3$

$$f'(x) = 2(3x^{3-1}) = 2(3x^2)$$

$$= 6x^2$$

Ex: find $g'(x)$ for $g(x) = \frac{4}{5x^2}$

$$g(x) = \frac{4}{5}x^{-2}$$

$$g'(x) = \frac{4}{5}(-2x^{-3}) = \frac{-8}{5}x^{-3}$$

$$= \frac{-8}{5x^3}$$

The Sum Rule

Let $f(x)$ and $g(x)$ be differentiable functions

$$(f(x)+g(x))' = f'(x)+g'(x)$$

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

Ex: Let $f(x)=3x^2+6x+1$ find $f'(x)$.

$f(x)=3x^2+6x^1+1$

$f'(x)=3(2x^1)+6(1x^0)+0$

$=6x+6$

Ex: $f(x) = \dfrac{x^3 + x^8}{x^4}$ find $f'(x)$

$$f(x) = \frac{x^3}{x^4} + \frac{x^8}{x^4} = \frac{1}{x} + x^4$$

$$= x^{-1} + x^4$$

then

$$f'(x) = -1x^{-2} + 4x^3$$

$$= \frac{-1}{x^2} + 4x^3$$

The Difference Rule

$f(x)$ and $g(x)$ are differentiable.

$$(f(x)-g(x))' = f'(x) - g'(x)$$

$$\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x))$$

Ex: Find an equation for the tangent line to the graph of $k(x)=5x^3-4x^2$ at $x=2$.

Point $k(2)=5(2)^3-4(2)^2=24$ $(2,24)$

Slope $k'(2)$

$k'(x)=5(3x^2)-4(2x)=15x^2-8x$

$k'(2)=15(2)^2-8(2)=44$ $m=44$

tangent line

$y-24=44(x-2)$

$y=44x-64$